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*Development of a Parallelizable Implementation of the Vorticity Transport Model*

**Abstract:**

The Vorticity Transport Model (VTM) modifies the Navier-Stokes equation and solves for the unsteady transport equation for vorticity. It then discretizes and solves this equation along with the Biot-Savart relationship to conservatively advect vorticity throughout a domain. A significant component of the algorithm involves domain decomposition using an octree structure. The octree provides for recursive grid refinement as well as a useful context for implementation of a Fast Multipole Method (FMM) for the purposes of calculating the Biot-Savart relationship. It is proposed that the octree domain structuring provides a ready-made framework for the implementation of a parallelizable computational approach that should provide significant scalability on cluster and grid computational platforms.

**Introduction:**

The Vorticity Transport Model (VTM) [1] recasts the Navier-Stokes equation into vorticity-velocity form

 (1)

Where the velocity field is determined from the vorticity field following the Biot-Savart law

 (2)

In the VTM Eqn. 2 is solved using a Cartesian fast multipole method (FMM) [2]. The FMM solves the n-body problem posed by Eqn. 2 much faster than a naïve direct solution by grouping multiple long-range interactions into one approximated bulk interaction [4]. The VTM is performed on a structured Cartesian grid that is adaptively refined to higher spatial definition as needed using an octree [2].

The VTM has been successfully used to simulate real-world phenomena [8]. However, it has faced computational challenges in satisfactorily performing adequate spatial refinement [9].

The structure and solution methodology inherent in the VTM lends itself quite nicely to solution via parallel means on a cluster or grid computing platform. The octree domain decomposition provides a powerful heuristic for parallelization [7]. Additionally, work has been done with the FMM and parallelization there as well [6].

The proposed research seeks to leverage the previous work done in parallelization of the various methods employed in the VTM and modify them to work within its framework. The hope is that this will enable significant scalability on scientific computing platforms and enable greater available fidelity in the resulting simulations.

**Technical Approach:**

The project primarily consists of three phases. First, basic algorithmic development needs to occur to create potential parallel solutions to the individual pieces of the VTM. Prototyping of un-optimized code will occur. In the second phase the prototype code will be re-written and optimized for the target language/platform. In the third phase, performance testing of the final code will occur. While performance may have had small or even mid-scale testing for development purposes in phases one and two, phase three will seek to test at full-scale.

Phase one will require several (2-3) algorithm developers to prototype code. The developers will need to have experience both in fluid mechanics, as well as CFD; particularly implementation of parallel codes.

Two potential development paths are available to pursue. Firstly, the current available code developed by Brown et al. could be used as a base and refactored to incorporate the likely extensive changes required. This may also pose potential legal/financial challenges depending on the nature of how the code/intellectual property is handled. Alternatively, development can start from the ground-up generating an entirely separate code-base. This may be more time consuming, but it will give additional flexibility. Other than human capital, the only real resources required will be workstations for code development.

Phase two will require several general purpose coders in addition to the personnel from phase one. The additional coders should have experience in parallel programming, high-performance computing, and ideally prior experience programming for the targeted hardware platforms/architecture that the code will be optimized for. Mid-scale testing to target important areas for optimization will require either rented time on a nearby cluster, or rented time on on-demand computing platforms (e.g. Amazon EC2)

Phase three will require only several of the original developers for testing purposes. Full-scale testing will be performed on rented time at a large computational facility or perhaps more easily on a very large Amazon EC2 cluster. The computational power of the cluster is unimportant the primary variable of interest is the number of nodes in the cluster or processors in the grid. The intention is to investigate the scalability of the final code.

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